2018

(April)

MATHEMATICS

(Elective/Honours)

(Statics and Dynamics)

(GHS–41)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. (a) Find the magnitude and direction of the resultant of two forces P and Q acting at a point of a rigid body. 5

(b) The angle of inclination between two forces P and Q is θ. If P and Q be interchanged in position, show that the resultant will be turned through an angle ϕ, where

$$\tan\frac{\phi}{2}=\frac{P-Q}{P+Q}\tan\frac{\theta}{2}$$ 5

(c) Forces acting at a point are represented in magnitude and direction by $2AB$,

(Turn Over)

3BC, 2CD, DA, CA and DB, where ABCD is a quadrilateral. Show that the forces are in equilibrium. 5

2\. (a) If the two like parallel forces P and Q $(P > Q)$ acting on a rigid body at A and B be interchanged in position, show that the point of application of the resultant be displaced along AB through a distance d, where

$$d = \frac{P - Q}{P + Q} \cdot AB$$ 5

(b) Show that the moment of a force about a point is equal to the algebraic sum of the moments of its components about that point. 5

(c) Show that any number of coplanar couples acting on a body is equivalent to a single couple whose moment is equal to the algebraic sum of the moments of the couples. 5

UNIT—II

3\. (a) Forces proportional to 1, 2, 3, 4 act along the sides AB, BC, AD, DC respectively of a square $ABCD$, the length of whose sides is 2 ft. Find the magnitude and the line of action of the resultant. 3+3=6

(Continued)

(b) A heavy uniform rod is in equilibrium with one end resting against a smooth vertical wall and the other against a smooth plane inclined to the wall at an angle θ. Prove that if α be the inclination of the rod to the horizon, then

$$\tan\alpha = \frac{1}{2}\tan\theta$$ 5

(c) State the laws of statical friction. 4

4\. (a) A uniform ladder rests with one end on the rough horizontal ground and the other against a rough vertical wall. The coefficient of friction at the lower and upper ends are $\frac{3}{7}$ and $\frac{1}{3}$ respectively. Determine the angle which the ladder makes with the ground when it is about to slip. 6

(b) Define centre of gravity (c.g.). Prove that the c.g. of a body is unique. 1+2=3

(c) Find the c.g. of a uniform trapezium lamina. 6

UNIT—III

5\. (a) A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to

(Turn Over)

infinity. Prove that the time it takes to reach a height h is

$$\frac{1}{3}\sqrt{\frac{2r}{g}}\left[\left(1+\frac{h}{r}\right)^{3/2}-1\right]$$

where r is the radius of the earth. 6

(b) A particle moves in a straight line, starting from rest at a distance a, towards the centre of force. If its acceleration at a distance x from the centre of force be $\mu / x^{5/3}$, show that it will reach the origin after a time

$$\frac{2a^{4/3}}{\sqrt{3\mu}}$$

5

(c) Two spheres of masses M, m impinge directly when moving in opposite directions with velocities u, v respectively. If the sphere of mass m is brought to rest by the collision, show that $v(m-eM) = M(1+e)u$. 4

6. (a) Two smooth imperfectly elastic spheres of masses m_1 and m_2 collide obliquely with velocities u_1 and u_2 making angles α_1 and α_2 with the line of centres. Calculate the loss in kinetic energy due to the impact. 6

(b) A particle rests in equilibrium under the attraction of two centres of force which attract directly as the distance, their intensities being μ, μ'. The particle is displaced slightly towards one of them. Show that the time of small oscillation is

$$\frac{2\pi}{\sqrt{\mu+\mu'}}$$

4

(c) In an SHM of period $\frac{2\pi}{\omega}$, if the particle be projected with a velocity u_0 from a point at a distance x_0 from the centre (away from the centre), prove that amplitude is

$$\left[x_0^2+\frac{u_0^2}{\omega^2}\right]^{1/2}$$

5

UNIT—IV

7. (a) A particle, of mass m, is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle be released from rest, show that the distance fallen through in time t is

$$g\frac{m^2}{\mu^2}\left[e^{-\frac{\mu t}{m}}-1+\frac{\mu t}{m}\right]$$

5

(b) A particle of mass m moves from rest in a straight line under the action of a constant force in a medium whose resistance to the motion is $m(a+bv)$, where a and b are constants and v is the velocity at time t. If V be the terminal velocity, prove that the particle in time t has moved a distance x, where

$$bx = V(bt - 1 + e^{-bt})$$ 5

(c) A particle is projected from a point on the ground level and its height is h when it is at the horizontal distances a and $2a$ from its point of projection. Prove that the velocity of projection u is given by

$$u^2 = \frac{g}{4}\left[\frac{4a^2}{h} + 9h\right]$$ 5

8. (a) A particle is projected vertically upwards with a velocity u against a resistance proportional to the square of the velocity. If V is the terminal velocity of the body and m its mass, show that, when the body has fallen back to the point of projection, the loss of kinetic energy is

$$\frac{1}{2}mu^2\left(\frac{u^2}{V^2+u^2}\right)$$ 6

(b) A ball is projected so as to just clear two walls, the first of height a at a distance b from the point of projection and the second of height b at a distance a from the point of projection. Show that the range on the horizontal plane is

$$\frac{a^2+ab+b^2}{a+b}$$

and the angle of projection exceeds $\tan^{-1} 3$. 5

(c) If t be the time in which a projectile reaches a point P in its path and t' the time from P till it reaches the horizontal plane through the point of projection, show that the height of P above the horizontal plane is $\frac{1}{2}gtt'$. 4

UNIT—V

9. (a) A point P describes, with a constant angular velocity ω about O, the equiangular spiral $r = ae^{\theta}$, O being the pole of the spiral. Obtain the radial and transverse accelerations of P. 4

(b) A shell lying in a straight smooth horizontal tube suddenly breaks into two portions, of masses m_1 and m_2. If s

is the distance apart in the tube of the masses after time t, show that the work done by the explosion is

$$\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} \cdot \frac{s^2}{t^2}$$ 5

(c) A particle is projected along the inner surface of a smooth vertical circle of radius a, its velocity at the lowest point being $\frac{1}{5}\sqrt{95ag}$. Show that it will leave the circle at an angular distance $\cos^{-1}\left(\frac{3}{5}\right)$ from the highest point and that its velocity then is $\frac{1}{5}\sqrt{15ag}$. 6

10. (a) A point moves along the arc of cycloid in such a manner that the tangent at its rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude. 4

(b) Show that all forces which are one-valued functions of distances from fixed points are conservative forces. 5

(c) Show that for a particle, sliding down the arc and starting from a cusp of a smooth cycloid whose axis is vertical and vertex lowest, the vertical velocity is maximum when it has described half the vertical height. 6

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